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# Overloading Prediction in Symmetric Cross Coupled Low-Pass Sigma Delta Modulators

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**Abstract**

This paper evaluates the prediction of stability of various symmetric cross coupled low-pass sigma delta(ΣΔ) modulators, by using the describing function theory. The symmetric cross coupled low pass ΣΔ modulator, enhances performance and stability when compared to conventional noise shaping sigma delta modulators and achieves up to 2Nth order noise shaping from N-path cross coupling with better in-band noise rejection without sacrificing conversion rate and bandwidth. Here, over-loading levels of symmetric cross coupled ΣΔ modulatorsare predicted and compared with time domain simulations for sinusoidal signals. The relationship between stability, NTF and quantizer levels has also been investigated.


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*Keywords:*analog-to-digital converter;cross coupled;noise shaping; over-loading; sigma-delta

## 1. Introduction

The applications of sigma-delta modulators (ΣΔ) have increased many fold in the past few years, especially in areas of high data rate and resolution requirements. To realize ΣΔ at high bandwidth, a low over sampling ratio (OSR) is preferred. Time-interleaved (TI) modulators achieve high signal-to-noise ratios (SNR) at low order and low OSRs[1].

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Several ADCs may be operated in parallel, using different clock phases, in the $N$-path sigma-delta modulator, where each path runs at 1/N times the overall sampling frequency ($f_s/N$), where $N$ is the number of paths used[2].The $N$-path architecture achieves the same SNR of single-path architecture with a lower order loop filter with benefits in terms of stability and complexity. A wideband cross-coupled sigma-delta modulator that uses this method to increase SNR and signal bandwidth has been analyzed[3]. In addition, the effect of noise cross-coupling on time-interleaved sigma-delta ADCs has been investigated[4]. Hamidi *et al*. proposed un-extended and extended noise-coupling techniques in TI cross-coupled ΣΔ modulators to improve the effective order to $(2N-1)$[5]. However, the stability of these modulators, with respect to quantizer overloading, has not been analysed to a great extend due to the highly non-linear nature of the quantizer. It has been proved that the stability of quantizer gets jeopardised at high input amplitudes, thus affecting the stability of the entire sigma delta modulator[6]. This is termed modulator over-loading, and the signal at which this occurs is known as the over-loading level.

A dual extended cross coupled sigma-delta modulator with improved noise shaping has been proposed[7]. Output spectra, for two path and three path shows improved performance. However, a study on its stability has not been made. This paper uses the multiple – input describing function[8,9] to predict its over-loading levels and thus improve the stability of the entire re-configurable modulator.

The rest of this paper is structured as follows: in Section 2& 3, we review the dual extended cross coupled sigma delta modulator and the describing function analysis of a quantizer. In Section 4, we apply the describing function to predict the over loading levels and present results and discussions based on time domain simulations and in Section 5, we present the conclusions.

## 2. The dual extended cross coupled Sigma Delta Modulator

The dual extended cross coupled sigma delta modulator, depicted in Fig.1[7], shows symmetric paths of the architecture. Each path is a first order sigma delta modulator, consisting of a linear filter and a nonlinear quantizer. The input-output relation of a quantizer shown in Fig. 2 can be mathematically represented as[12]

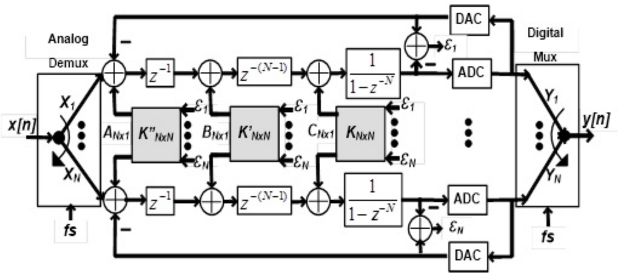


Fig.1.Dual extended cross coupled Sigma Delta Modulator

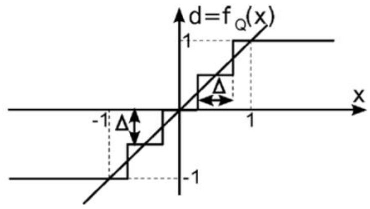


Fig. 2.Input – Output relation of Quantizer

$$Q(x) = u(x)\sum_{k=0}^{\frac{n}{2}-1} \Delta u\left(x-\frac{\Delta}{2}k\Delta\right) - \; u(-x)\sum_{k=0}^{\frac{n}{2}-1} \Delta u\left(-x-\frac{\Delta}{2}k\Delta\right) \quad (1)$$

where $\Delta$ denotes the quantization step and *n* the number of discontinuities in the quantization function. With this definition of *n*, the number of quantization levels $n_{lev}$ equals n+1. In a practical unit-element DAC implementation n would correspond to the number of unit-elements. The function *u(x)* is the standard step function

$$u(x) = \begin{matrix} 0 & x<0 \\ 1 & else \end{matrix} \quad (2)$$

The equation holds true when n is even and represents a "mid-thread" quantizer showing that the quantizer is a non-linear device. The quantizer is normalized to have a gain of 1 and a full scale input range of [-1 : 1], hence

$$\Delta = \frac{2}{n} \tag{3}$$

## 3. Describing Function analysis of quantizer

Describing function analysis[8, 9] is an approximation for analyzing nonlinearity by quasi-linearization, which is the approximation of the non-linearity by a linear system that depends on the amplitude of the input[8]. The open literature has comparatively little reports on non-linearity of multi-bit ΣΔ modulators, which may be accurately modeled by a linear gain and an additive white noise[9-10]. It is also shown that the multi-bit quantization is non-linear and has a limiting overloading level[11]. The accurate prediction of stability limits of the multi-bit quantizer remains to be a problem.

The quantizer gain is determined simultaneously by both the statistical properties of the signal and quantization noise variance at the quantizer input, given by[6]

$$K = \frac{\Delta}{\sigma_{eq}} \frac{1}{\sqrt{1+\rho^2}} \sqrt{\frac{2}{\pi}} \tag{4}$$

It is assumed that the input x(t) of the quantizer consists of two orthogonal components; the noise component assumed to be Gaussian, $x_n(t)$ and the signal component, $x_s(t)$. Linear gains $K_N$ and $K_S$ are associated with the noise and signal components respectively. This linearization process introduces a linearization error $Q_Q$ defined as

$$Q_Q(x) = Q(x) - (KsXs + KnXn) \tag{5}$$

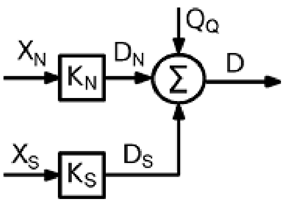


Fig.3. The dual input describing function model of quantizer

Fig. 3 depicts quasi-linear modelling of quantizer. As it has been proved that both $K_N$ and $K_S$ depends on the signal, on including such a model in the feed-back loop of a sigma delta modulator, there will exist a value of input signal for which the loop becomes unstable. Considering a sinusoidal input with Amplitude *Ain*, the signal component of the modulator output is also sinusoidal and will be of the value.

$$Ds = STF(Ks)Vin \tag{6}$$

Since the loop gain in the signal band is large, the STF may be assumed to be close to unity, and thus, $Ds = Vin$ may be assumed. Hence,

$$X_N = \frac{-H(z)}{1 + H(z)K_N} Q_Q \quad (7)$$

$$X_S = \frac{D_s}{K_S} = \frac{V_{in}}{K_S} \quad (8)$$

$$Q_Q(x) = D - KnXn - KsXs \quad (9)$$

An iterative algorithm to obtain a numerical solution to these unknown quantities, namely $K_N, K_S, X_S, X_N$ $and$ $Q_Q$ has been presented[12] along with a toolbox in the open domain.

The describing function analysis was performed on symmetric dual extended cross coupled sigma delta modulators along with the extended and un-extended symmetric cross coupled sigma delta modulators by extracting their NTF values. Over loading values have been predicted and compared with mathematical time-domain based simulations and architectural time domain simulations in SIMULINK® and MATLAB®.

## 4. Simulation Results and Discussions

The NTF based on architecture was first determined. For a dual extended symmetric cross coupled sigma delta modulator, an NTF for out-of band noise gain of 8 was obtained as

$$NTF = \frac{(z-1)^4}{(z^2 - 0.433z + 0.06031)(z^2 - 0.06375z + 0.1968)} \quad (10)$$

With the mid-thread quantizer assumed, the describing function analysis was performed to predict an overloading level of 0.56. Further, upon time domain simulations using the architectural model, the modulator was seen to overload at 0.57, proving the predictions true to a high degree. Simulations with the same parameters were performed on extended and conventional architectures, varying out-of band noise gain from 1 to maximum attainable value of 4 for conventional architecture, 8 for extended architecture and 16 for the dual extended architecture.

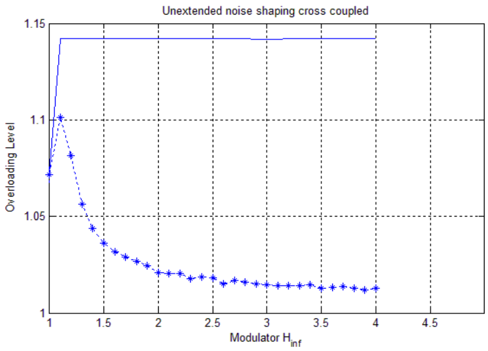


Fig.4. Prediction and overloading simulation in un-extended symmetric cross coupled architecture

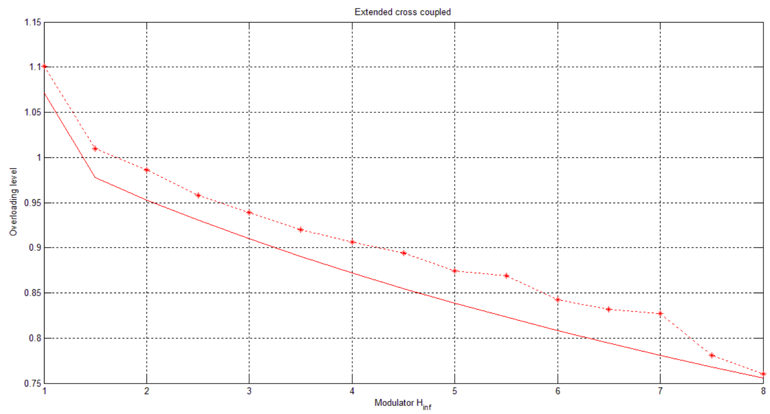


Fig. 5. Prediction and overloading simulation in extended symmetric cross coupled architecture

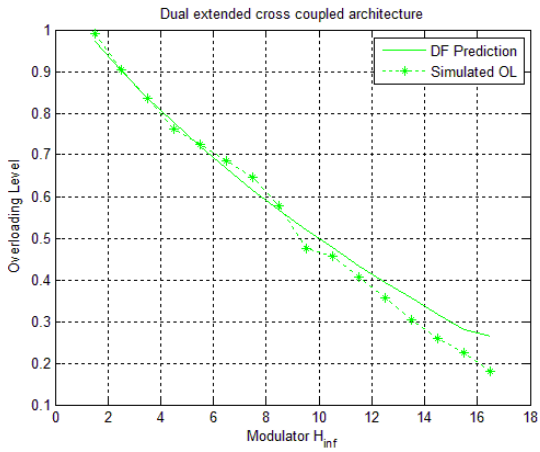


Fig.6. Prediction and overloading simulation in dual extended symmetric cross coupled architecture

The simulations results shown in Fig. 4 shows the DF predicted value and time domain simulation values in a conventional sigma delta modulator, which corresponds to an effective order of 2. As seen, the values do not correlate satisfactorily. Fig. 5 shows the DF predicted value and time domain simulation values in an extended symmetric sigma delta modulator, which corresponds to an effective order of 3. Here in, the correlation between DF predicted values and time domain based behavioral simulations exhibit a higher level.

Fig. 6 shows the DF predicted value and time domain simulation values in the dual extended sigma delta modulator, which corresponds to an effective order of 4. Here, a high level of correlation between DF predicted values and time domain based behavioral simulations exhibits a very highcorrelation of 0.9969.

Fig. 7 shows a comparison of all three architectures predictions and simulated overloading values.

In Fig. 7, it may be noted that in the case of un-extended symmetric sigma delta modulator, the predictions and simulation values do not vary beyond an out-of-band gain of 4, for extended symmetric sigma delta modulator, an out-of band gain of 8 and for a dual extended symmetric sigma delta, an out of band gain of 16. This agrees to the theory that the maximum attainable $H_{inf}$ for a second order NTF is 4, for third order is 8 and for forth order is 16.

Further investigation was performed to check variation in prediction algorithms upon varying levels of the quantizer. A few of its results is tabulated in Table 1. The testing was performed for an Over Sampling Ratio fixed at 15, 2 MHz bandwidth and 1MHz input frequency with a sampling frequency of 60MHz and maximum amplitude fixed at 1.35V.

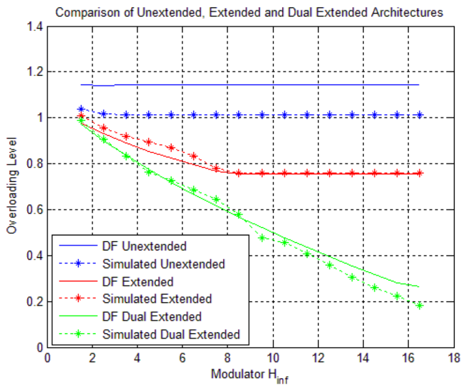


Fig. 7.Describing function predictions (marked with -) and simulated value (marked with *) with varying $H_{inf}$ for Un-extended, Extended and Dual Extended cross coupled ΣΔ modulators

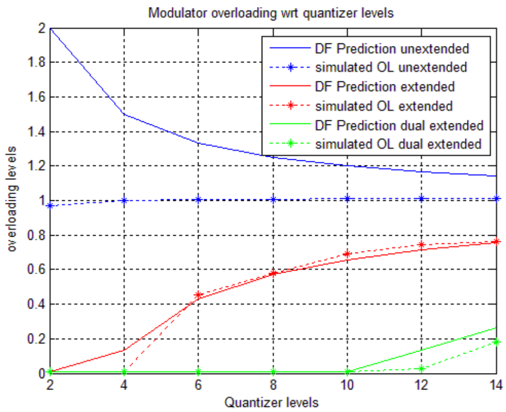


Fig. 8.Modulator overloading with respect to quantizer levels of un-extended, extended and dual extended cross coupled ΣΔ modulator

Table 1. Predicted over-loading levels and architectural time domain simulation results

| Type | Quantizer Levels | Input voltage for overloading | Predicted Value |
|---|---|---|---|
| 1. Dual extended 2 path cross coupled ΣΔ architecture | 17 | | 0.3549 |
| | 15 | 0.322 V | 0.2633 |
| | 13 | 0.183 V | 0.1308 |
| 2. Extended 2 path cross coupled ΣΔ architecture[6] | 15 | 0.809 V | 0.7556 |
| | 13 | 0.721 V | 0.7148 |
| | 11 | 0.598 V | 0.6578 |
| | 9 | 0.431 V | 0.5723 |
| | 7 | 0.145 V | 0.4315 |
| 3. Un-extended 2 path cross coupled ΣΔ architecture[4,6] | 15 | 1.17 V | 0.9994 |
| | 13 | 1.14 V | 0.9993 |
| | 11 | 1.08 V | 0.9992 |
| | 9 | 1.04 V | 0.9990 |
| | 7 | 0.936 V | 0.9993 |

From Table 1 and Fig. 8, it is observed that the dual extended symmetric cross coupled sigma delta modulator requires a 4 bit quantizer (2+2 bit pipelined architecture) to process an input signal with amplitude less than or equal to -12dBFS, against what was predicted by theory. Upon further investigation, it was observed that the extended noise shaping requires a 3.5 bit quantizer to process an input signal with amplitude less than or equal to -6dBFS and un-extended noise shaping requires a 2 bit quantizer to process an input signal with amplitude less than or equal to -6dBFS.

## 5. Conclusions

The limits of input amplitude to symmetric cross coupled sigma delta modulators have been predicted for a sinusoidal signal, by employing the describing function based quasi-linear modeling of the saturating quantizer. The predicted values show very close agreement with time domain based architectural simulations with a correlation of 0.9969 in the case of dual extended symmetric cross coupled sigma delta modulators. The correlation between predicted values and simulated values in the case for extended symmetric cross coupled sigma delta modulators was observed to be 0.9917. However, the relation in the case of un-extended sigma delta modulators was observed to be poor. From observations, it might be advisable to use the describing function based quasi-linear modeling prediction algorithm only for higher order modulators and not for lower order modulators.

The relationship between stability, NTF and quantizer levels has also been mathematically investigated and results have been reported. The results presented here may be utilized for an optimized design criteria for higher order sigma delta modulators for various applications. The prediction model may be extended to predict over-loading for any general input against the sinusoidal input considered in this paper. An accurate non-overloading symmetric cross coupled sigma delta modulator for high bandwidth multi-standard applications with reduced hardware compared to the traditional alternatives may be designed with the help of prediction algorithms, the result of which would be reported in future.

## References

1. LeeKS, Maloberti F. Time-interleaved sigma-delta modulatorusing output prediction scheme,*IEEE Trans. Circuits Syst. II, Exp.Briefs*2004;**51**(10), p.537–541.
2. BernardinisG.*et al.* A wide-band 280-MHz four-path time-interleaved bandpass sigma-delta modulator,*IEEE Trans. Circuits Syst. I, Reg. Paper*2006;**53**(7), p. 1423–1432.
3. Bilhan E, MalobertiF. A wide-band 2-path cross-coupledsigma delta ADC, *in Proc. IEEE Int. Symp. Circuits Syst.*, 2008; p. 1220–1223.
4. PamartiS. The effect of noise cross-coupling on time-interleaveddelta-sigma ADCs,*IEEE Trans. Circuits Syst. II, Exp. Briefs*2008, **55**(6), p. 532–536.
5. B. Hamidi and H. Miar,Extended Noise Shaping in Sigma-Delta Modulator Using Cross-Coupled Paths,*IEEE Trans. Circuits Syst. I, Reg. Papers*2014, **PP**(99) , p. 1-12.
6. Lota J, Al-Janabi M, KaleI. Nonlinear Stability Prediction of Multibit Delta-Sigma Modulators for Sinusoidal Inputs, *IEEE Trans. Instrumentation and Measurement*2014, **63.**(1).
7. Prakash J, BabithaR, JimsonM.A Low-Power Reconfigurable Cross-Coupled Σ-Δ Modulator for Multi-standard Wireless Applications, *Advances in Computing and Communications (ICACC), 2014 Fourth International Conference on*2014, p.130-133.
8. GelbA, Vander Velde WE. Multiple-Input Describing Functions and Nonlinear System Design. New York: McGraw-Hill; 1968.
9. Ardalan S, Paulos J. An analysis of nonlinear behavior in delta-sigma modulators, *IEEE Trans. Circuits Syst.* 1987, **34**(6), p. 593–603.
10. HeinS, ZakhorA. On the Stability of Σ-Δ modulators, *IEEE Trans. Signal Process*1993.**41**(7), p. 2322-2348.
11. Baird R, FiezT.Stability Analysis of higher order Δ-Σ modulation for ADCs, *IEEE Trans. Circuits and Systems II*, Analog Digit. Signal Process. 1994, **41**, (1), p. 59-62.
12. Rombouts P, Bock M, Maeyer J, Ludo Weyten. A describing function study of saturated quantization and its application to the stability analysis of Multi Bit Sigma Delta modulators,*IEEE Trans. Circuits and Systems-I, Reg. Papers*2013, **60** (7).